MATH 579 Unit 2 Exam Name:

Instructions:

Please write your answers on separate paper. Please write clearly and legibly, using a large font and plenty of white space (I need room to put my comments). Staple all your pages together, with your problems in order, when you turn in your exam. Please don't write under the staple. Make clear what work goes with which problem. Put your name or initials on every page. To get credit, you must show adequate work to justify your answers. If unsure, show the work. Simplify your answers, except you may leave any number of more than three digits (like 123×456 or $15^{\underline{4}}$) unsimplified. No outside materials are permitted on this exam – no notes, papers, books, calculators, phones, smartwatches, or computers – only pens and pencils, and your coursepack. You may use any result in the coursepack (whether boxed or an exercise). However, you must cite it, and you may not use it to prove itself (or a portion/special case of itself). Each problem is out of 10 points, 100 points maximum. You have 75 minutes.

1. Set $a_i = i4^i$. Find the OGF $A(x) = \sum_{i \geq 0} a_i x^i$.
2. Consider the OGF $A(x) = \frac{1-2x^3}{(1-x)^4} = \sum_{n \geq 0} a_n x^n$. Find a closed form for a_n. Simplify for 1pt of extra credit.
3. Let a_i satisfy $a_0 = 1, a_1 = 12, a_i = 6a_{i-1} - 9a_{i-2} + 1$ $(i \geq 2)$. Find the OGF.
4. Your habit in climbing stairs is to always go up either one or two steps at a time. Going up one step can be with either the left foot or with the right foot. Going up two steps can be with the left foot, the right foot, or jumping with both feet at once. Let $S(n)$ denote the number of ways to reach the n-th stair. Use OGFs to find a closed form for $S(n)$.
5. Let a_n denote the number of ways to color a $1 \times n$ chessboard using the colors red, white, and blue, so that a red square must be followed by a white square. Find the OGF representing this sequence.
6. Let $a \in \mathbb{N}_0$. Prove that $\sum_{i \geq 0} \binom{a}{2i} = \sum_{i \geq 0} \binom{a}{2i+1}$.
7. Use the multinomial theorem to count the number of distinct rearrangements of Ponomarenko.
8. Consider the sequence $a_0 = 1$, $a_n = na_{n-1} + n(n-1)$ (for $n \geq 1$). Find the EGF $A(x)$.
9. Let a_n denote the number of ways of splitting a list of n people (say, in alphabetical order), into sublists of size 4 or 5. Each sublist must consist of consecutive names. Find an appropriate generating function (either OGF or EGF) and use this to calculate a_{17}.
10. Let b_n denote the number of ways of partitioning a set of n people into groups of size 4 or 5. Find an appropriate generating function (either OGF or EGF) and use this to calculate b_{17}.